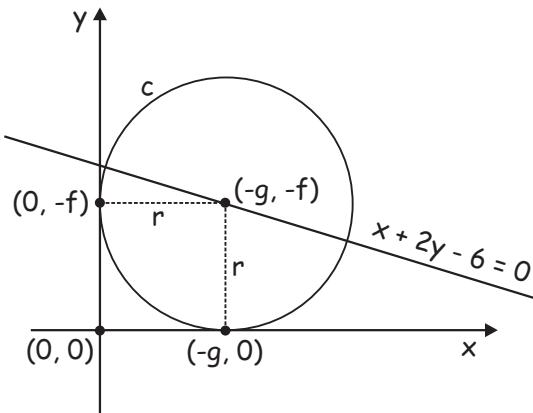


SAMPLE PAPER 2014: PAPER 2**QUESTION 4 (25 MARKS)**

Sketch the situation.

The circle c touches the x -axis at $(-g, 0)$ and touches the y -axis at $(0, -f)$.

The line passes through the centre of the circle c .



FORMULAE AND TABLES BOOK
Co-ordinate geometry: Circle [page 19]

Given centre (h, k) and radius r

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $(-g, -f)$

Radius $\sqrt{g^2 + f^2 - c}$

$(-g, -f) \in x + 2y - 6 = 0$ [If a point is on a line, substitute the point into the equation of the line and it satisfies the equation.]

$$-g - 2f - 6 = 0$$

$$g + 2f = -6 \dots\dots\dots(1)$$

The points $(-g, 0)$ and $(0, -f)$ satisfy the equation of the circle.

$$(-g, 0) \in c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore (-g)^2 + (0)^2 + 2g(-g) + 2f(0) + c = 0$$

$$g^2 - 2g^2 + c = 0$$

$$c = g^2$$

$$(0, -f) \in c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore (0)^2 + (-f)^2 + 2g(0) + 2f(-f) + c = 0$$

$$f^2 - 2f^2 + c = 0$$

$$c = f^2$$

$$\therefore c = g^2 = f^2 \dots\dots\dots(2)$$

$$\therefore g^2 = f^2 \Rightarrow g = \pm f \dots\dots\dots(3)$$

To find the equation of each circle, choose $g = +f$ for the first circle equation and $g = -f$ for the second circle equation.

$$g = +f \quad [\text{Substitute into Eqn. (1)}]$$

$$g + 2g = -6$$

$$3g = -6$$

$$g = -2$$

$$f = -2 \quad [\text{Substitute into Eqn. (2)}]$$

$$c = g^2 = (-2)^2 = 4$$

$$g = -f$$

$$-f + 2f = -6$$

$$f = -6$$

$$g = 6$$

$$c = g^2 = (6)^2 = 36$$

EQUATION OF CIRCLES

$$x^2 + y^2 + 2(-2)x + 2(-2)y + 4 = 0 \Rightarrow x^2 + y^2 - 4x - 4y + 4 = 0$$

$$x^2 + y^2 + 2(6)x + 2(-6)y + 36 = 0 \Rightarrow x^2 + y^2 + 12x - 12y + 36 = 0$$